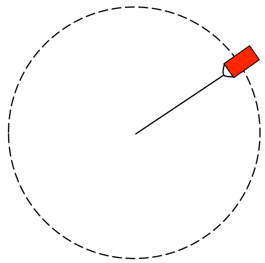
Problem 6.12

This is a relatively simple, classic problem. The pail is shown at an arbitrary point to the right.



a.) The two external forces acting on the pail are gravity and the normal force provided by the pail on the water. (Note that the tension force acts on the pale, not the water, so that force would not be included as the question is stated.)

b.) Which of the forces is most important in making the water move in a circular path?

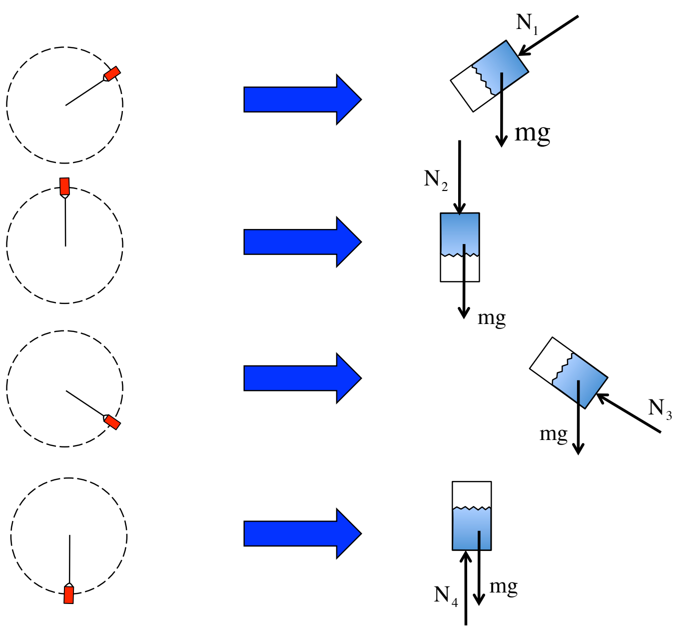
This is a bit of a trick question. Assuming that the normal force exerted by the pail on the water happens only from the bottom of the pail, consider the four pail-positions shown on the next page and the f.b.d.'s associated with each case:

Position A

Position B

Position C

Position D



Things to note:

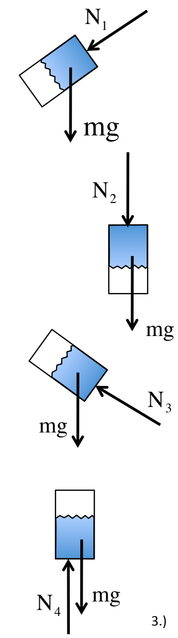
i.) The normal force, acting on the pail's bottom, always points toward the center of the arc. It is, in other words, fully a center-seeking, centripetal force.

ii.) Although the speed will be different at various positions, the normal force will also be different. That is, at the bottom of the arc it will have to be very large so as to overcome gravity downward and provides an additional mv^2/R force upward.

iii.) Gravity is always downward, almost always with only a component in the centripetal direction (in fact, when the pail is in the horizontal, there is no centripetal component of gravity at all).

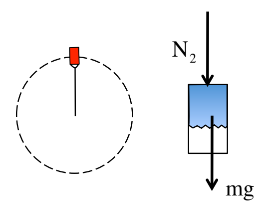
Conclusion? It is the normal force acting from the pail's bottom that is the most important centripetal force in the motion.

Minor point: If only gravity acted, the body would *not* follow a circular path but, rather, a parabolic path.



c.) What's the minimum velocity for the water to stay in the bucket when at the top?

A sketch of the system and the appropriate f.b.d. for the forces on the water are shown to the right. Keeping track of the signs (I'm talking positive to be "up") and using N.S.L. for the most general case (that is, for any velocity that will work), we can write:



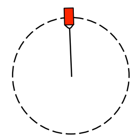
$$\sum F_c :$$

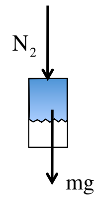
$$\Rightarrow \quad -N - mg = -ma_c$$

$$\Rightarrow \quad N + mg = m\left(\frac{v_{max}^2}{R}\right)$$

$$\Rightarrow \quad v_{max} = \left(\frac{(N+mg)R}{m}\right)^{1/2}$$

The trick is to notice that when the pail is passing through the top of the arc moving as slowly as possible, the normal force will go to zero and all of the centripetal force will be provided by gravity. That is:





$$v_{max} = \left(\frac{(\overset{0}{\cancel{N}} + mg)R}{m} \right)^{1/2}$$

$$\Rightarrow \quad v_{max} = (gR)^{1/2}$$

$$= \left[(9.80 \text{ m/s}^2)(1.00 \text{ m}) \right]^{1/2}$$

$$= 3.13 \text{ m/s}$$

d.) If the pail suddenly disappeared at the top of the arc moving at the velocity determined in *Part c*, what would happen to the water?

At the top of the arc, the pail and water would be moving with velocity:

$$\vec{v}_{top} = (3.13 \text{ m/s})\hat{i} + 0\hat{j}.$$

If the pail suddenly disappeared at the top, given that velocity, the water would do exactly what the pail would have done if the rope had been cut at that point. It would follow the parabolic arc expected of any freefalling object that was acted upon solely by gravity. The sketch to the right shows it all.

$$\vec{v}_{top} = (3.13 \text{ m/s})\hat{i} + 0\hat{j}$$